TeXsor3DG
An Explicit 3D Cartesian Discontinous Galerkin Spectral Element Compressible Navier-Stokes Solver

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Motivation

High order methods in aerodynamics

Higher accuracy with fewer degrees of freedom

Fewer elements needed

Nodal Discontinuous Galerkin finite elements

Cartesian Setting-Higher efficiency per degree of freedom

Governing Equations

Compressible Navier-Stokes equations

$$\frac{\partial U_m}{\partial t} + \frac{\partial F_{mi}}{\partial x_i} = 0$$

Conservative variables $U = \{\rho, \rho u, \rho v, \rho w, \rho E\}^T$

$$F = \left\{ \begin{array}{ccc} \rho u & \rho v & \rho w \\ \rho u^2 + P - \tau_{11} & \rho uv - \tau_{12} & \rho uw - \tau_{13} \\ \rho uv - \tau_{21} & \rho v^2 + P - \tau_{22} & \rho vw - \tau_{23} \\ \rho uw - \tau_{31} & \rho vw - \tau_{32} & \rho w^2 + P - \tau_{33} \\ \rho uH - \tau_{1j} u_j + q_1 & \rho vH - \tau_{2j} u_j + q_2 & \rho wH - \tau_{3j} u_j + q_3 \end{array} \right\}$$

$$\rho E = \frac{P}{\gamma - 1} + \frac{1}{2}\rho(u^2 + v^2 + w^2)$$

DG Formulation

Multiply by test function and integrate

$$\int_\Omega \Psi_r \left(\frac{\partial U_m}{\partial t} + \frac{\partial F_{mi}}{\partial x_i} \right) \mathrm{d}\Omega = \int_\Omega \Psi_r S_m \mathrm{d}\Omega$$

Integrate by parts

$$R_{mr} = \int_\Omega \left(\Psi_r \frac{\partial U_m}{\partial t} - \Psi_r S_m - \frac{\partial \Psi_r}{\partial x_i} F_{mi} \right) \mathrm{d}\Omega + \int_\Gamma \Psi_r F_{mi} \mathrm{n}_i \mathrm{d}\Gamma = 0$$

Inviscid flux: Lax-Friedrichs and Roe

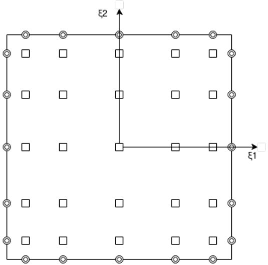
Viscous flux: symmetric interior penalty (SIP)

Tensor Basis Functions

Letting $\Psi_{ijk} = \phi_{\xi_i^1}\phi_{\xi_j^2}\phi_{\xi_k^3}$ for $i, j, k = 0, \cdots, M$ and solution expansion coefficients a, written as:

$$U_m(\xi, t) = \sum_{k=0}^{M}\sum_{j=0}^{M}\sum_{i=0}^{M} a_{ijk}(t)\phi_{\xi_i^1}\phi_{\xi_j^2}\phi_{\xi_k^3}$$

Let $\phi_{\xi_i} = \mathcal{L}_i$, the 1-D Lagrange polynomial using the Gauss-Legendre quadrature points.



Solver Capabilities

- Nodal Discontinuous Galerkin Finite Elements
  - solution coefficients are the solutions at the quadrature points – mass matrix is **diagonal**

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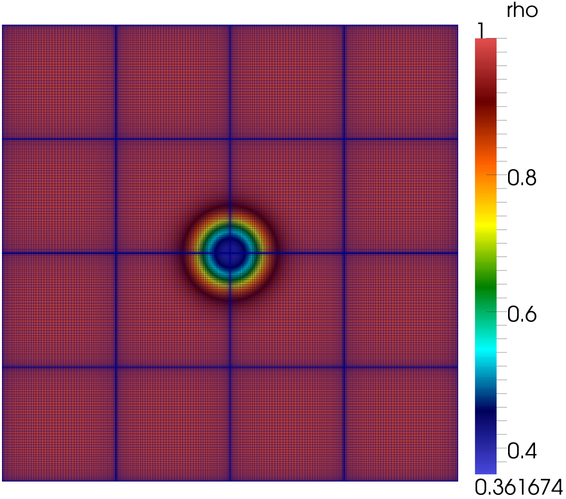
- Polynomial degree of p=63
  - Restricted by CPU RAM on computing nodes (p=15)

64th Order Solution



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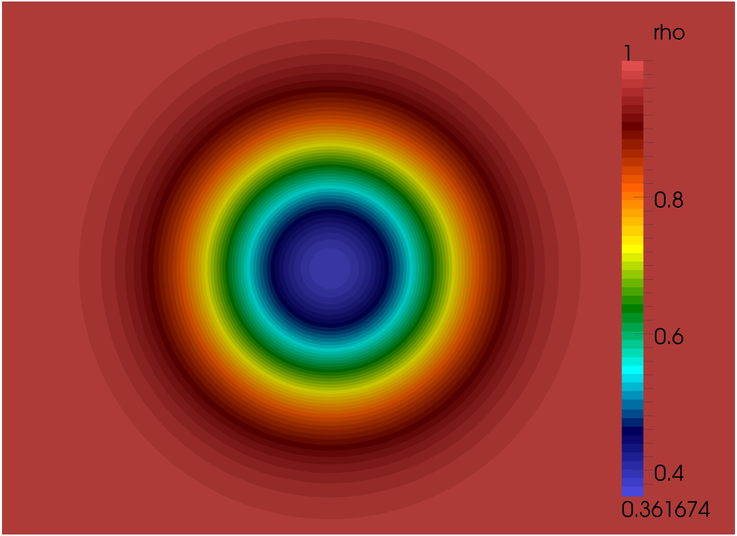

Solver Capabilities Cont.

- Explicit time stepping through Method of Lines
  - Forward Euler
  - 4th-order explicit Runge-Kutta
  - Low-storage Runge-Kutta 3rd Order (near future)

[1] Design of a Variational Multiscale Method for Turbulent Compressible Flows

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  - * explicit time step consistent with Laslo Diosady and Scott Murman[1]

$$
\begin{aligned}
\delta t &= min\left(\frac{S * h}{4\left(\|U\| + c\right)}, \frac{h^2}{\nu}\right) \\
h &= \frac{min(dx, dy, dz)}{(p+1)^{2.5}} \\
S &= \text{Number of Stages}
\end{aligned}
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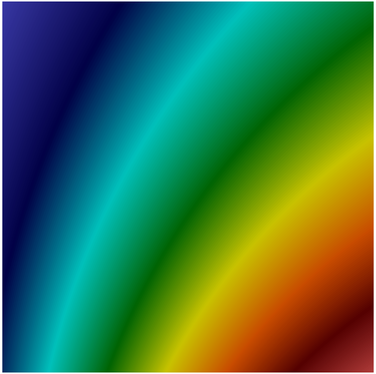
- Parallel through MPI

[1] Design of a Variational Multiscale Method for Turbulent Compressible Flows

Validation Results: Ringleb Flow

Problem Description[2]

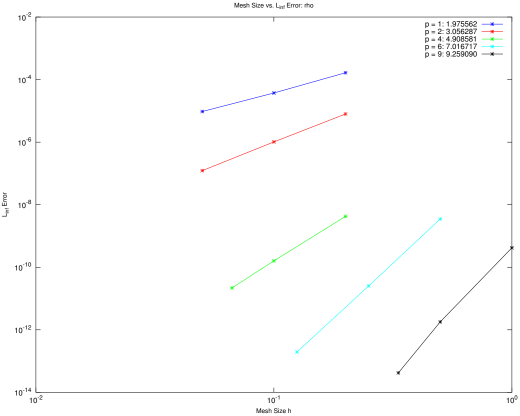
Governing equations: 2D Euler equations with $\gamma = 1.4$



[2] http://www.as.dlr.de/hiocfd/case_c1.2.pdf

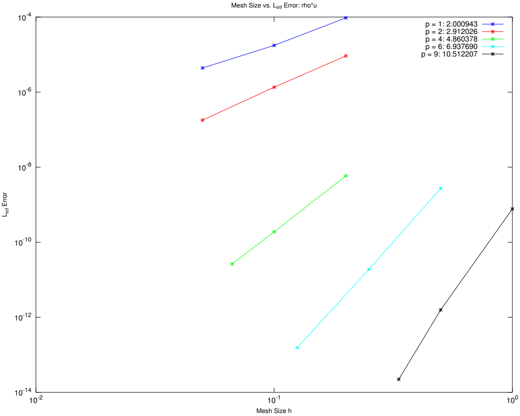
Validation Results: Ringleb-Rho

P	1	2	4	6	9
Slope	1.975562	3.056287	4.90858	7.016717	9.259090



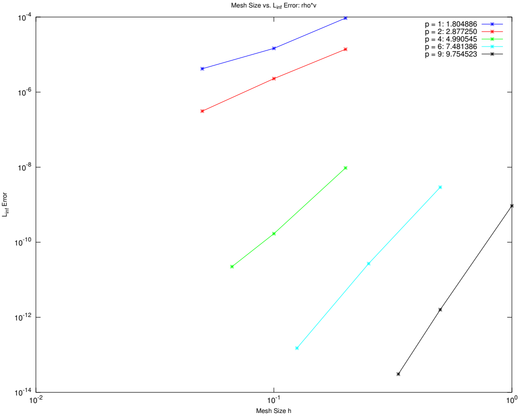
Validation Results: Ringleb-RhoU

P	1	2	4	6	9
Slope	2.000943	2.912026	4.860378	6.937690	10.51220



Validation Results: Ringleb-RhoV

P	1	2	4	6	9
Slope	1.804886	2.877250	4.990545	7.481386	9.754523



Validation Results: Ringleb-RhoE

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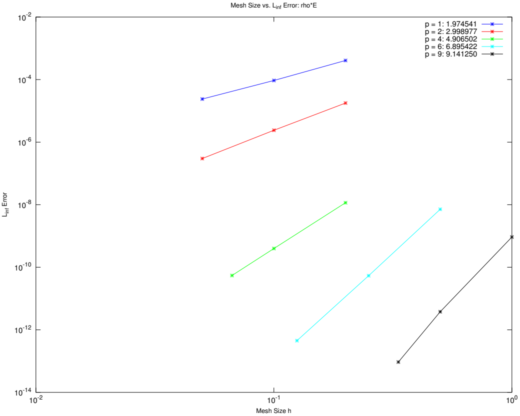
P	1	2	4	6	9
Slope	1.974541	2.998977	4.906502	6.895422	9.141250



Validation Results: Taylor Green Vortex

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Problem Description[3]

Domain: $[-\pi L, \pi L]^3$

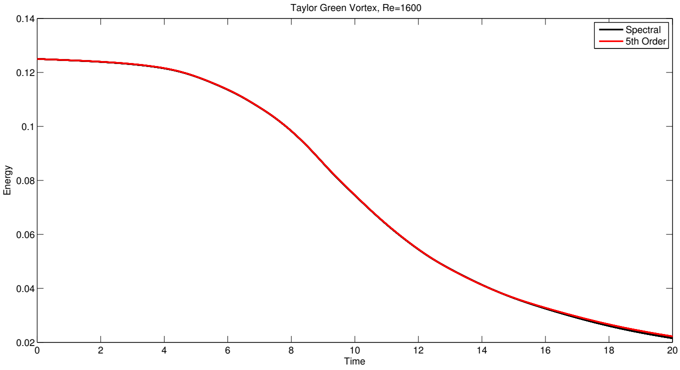
$$
\begin{aligned}
M_0 &= 0.1 \\
Re &= 1600 \\
Pr &= 0.71 \\
u &= V_0 \sin(x/L)\cos(y/L)\cos(z/L) \\
v &= -V_0 \cos(x/L)\sin(y/L)\cos(z/L) \\
w &= 0 \\
p &= \rho_0 V_0^2 \left[\frac{1}{\gamma M_0^2} + \frac{1}{16}\left(\cos(2x) + \sin(2y)\right)\left(\cos(2z) + 2\right)\right]
\end{aligned}
$$

Mesh: 64x64x64

p = 4 (5th order)

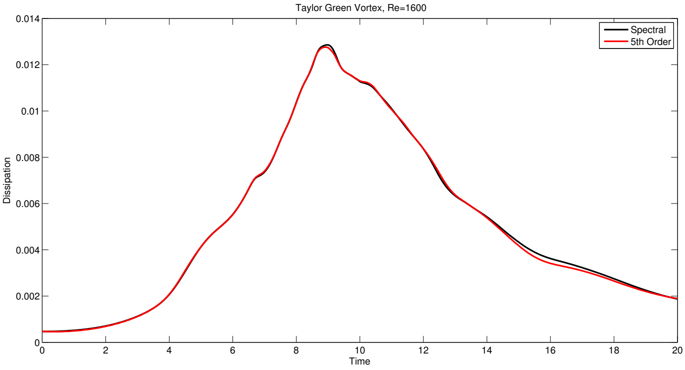
[3] http://www.as.dlr.de/hiocfd/case_c3.5.pdf

Validation Results: Taylor Green Vortex



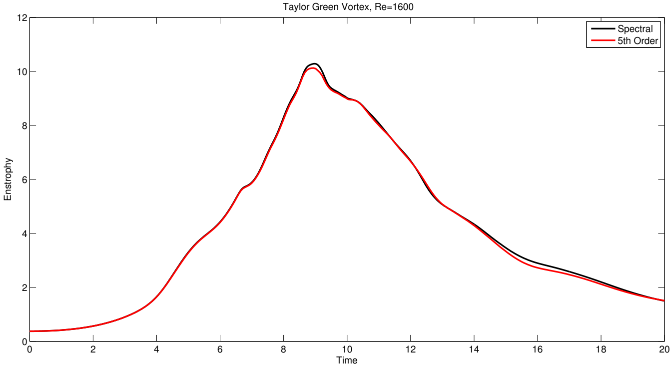
Energy

Validation Results: Taylor Green Vortex



Dissipation

Validation Results: Taylor Green Vortex



Enstrophy

Timing Results

All results are computed in serial on Intel Xeon E5-2670 processors with a clock speed of 2.6Ghz and 2GB per core memory.
• Mount Moran TAU benchmark = 7.2 sec

Mount Moran Specs:
• 93.92 Tflops cluster serving the University of Wyoming
• 218 nodes with 2, eight-core Intel Xeon E5-2670 Sandy Bridge processors on each node (3,488)

Timing Results: 3D Euler

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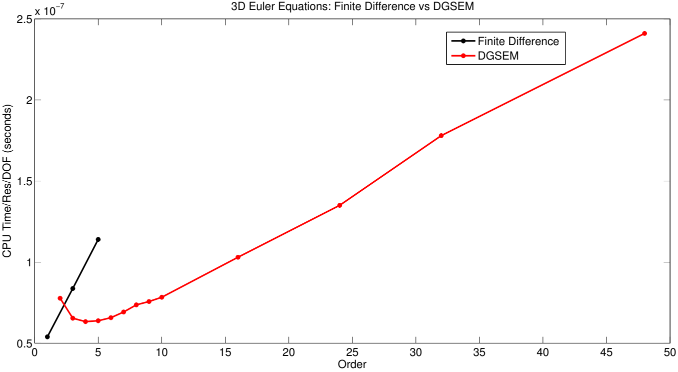
DOF=(number of fields)$(p+1)^3 NxNyNz$

Three-dimensional Euler Equations

Code	Order	DOF	Mesh Size	Time/Res/DOF
Finite Difference	1	5,151,505	100 x 100 x 100	5.39e-8
Finite Difference	3	5,151,505	100 x 100 x 100	8.37e-8
Finite Difference	5	5,151,505	100 x 100 x 100	1.14e-7
DGSEM	2	5,000,000	50x50x50	7.77e-8
DGSEM	3	4,851,495	33x33x33	6.54e-8
DGSEM	4	5,000,000	25x25x25	6.33e-8
DGSEM	5	5,000,000	20x20x20	6.38e-8
DGSEM	6	5,306,040	17x17x17	6.57e-8
DGSEM	7	4,705,960	14x14x14	6.92e-8
DGSEM	8	4,423,680	12x12x12	7.36e-8
DGSEM	9	4,851,495	11x11x11	7.57e-8
DGSEM	10	5,000,000	10x10x10	7.83e-8
DGSEM	16	4,423,680	6x6x6	1.03e-7
DGSEM	24	4,423,680	4x4x4	1.35e-7
DGSEM	32	4,423,680	3x3x3	1.78e-7
DGSEM	48	552,960	1x1x1	2.41e-7

Timing Results: 3D Euler



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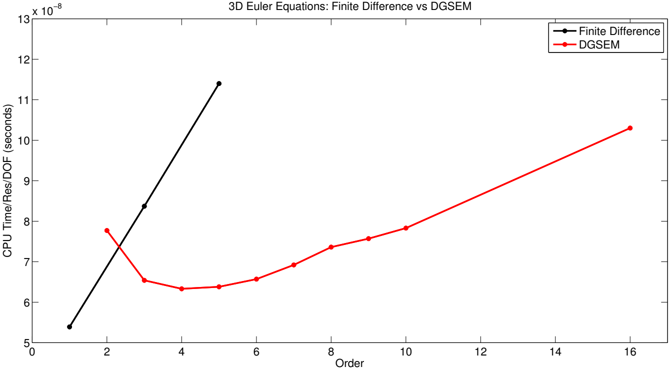

At 5th order, DGSEM is ∼twice as efficient!

Timing Results: Cartesian 3D Navier-Stokes

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DOF=(number of fields)$(p+1)^3 NxNyNz$

Three-dimensional Compressible Navier-Stokes Equations

Code	Order	DOF	Mesh Size	Time/Res/DOF
DGSEM	2	5,000,000	50x50x50	2.74e-7
DGSEM	3	4,851,495	33x33x33	2.59e-7
DGSEM	4	5,000,000	25x25x25	2.45e-7
DGSEM	5	5,000,000	20x20x20	2.68e-7
DGSEM	6	5,306,040	17x17x17	2.97e-7
DGSEM	7	4,705,960	14x14x14	3.25e-7
DGSEM	8	4,423,680	12x12x12	3.30e-7
DGSEM	9	4,851,495	11x11x11	3.63e-7
DGSEM	10	5,000,000	10x10x10	3.87e-7
DGSEM	16	4,423,680	6x6x6	8.30e-7

Timing Results: Cartesian 3D Navier-Stokes

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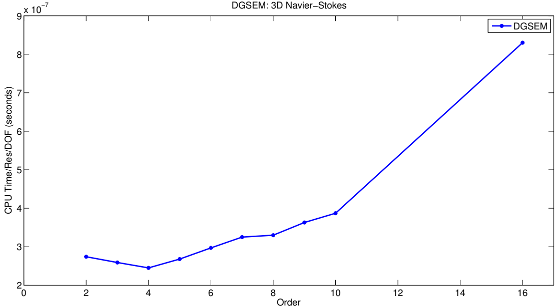

Laslo Diosady and Scott Murman $\approx 1.5e^{-7}$ (**non-Cartesian**) [4]
F. Hindenlang, G. Gassner $\approx 4.0e^{-7}$ (**non-Cartesian**) [5]

[4] Design of a Variational Multiscale Method for Turbulent Compressible Flows
[5] Explicit Discontinuous Galerkin methods for unsteady problems

Results: Parallel Scalability

- Strong Scalability
- Computed on Mount Moran and Yellowstone
- Taylor-Green Vortex

MPI Implementations:

- MPI Cartesian Topology
- MPI Derived Data Types
  - MPI_Type_Contiguous (x-y plane faces)
  - MPI_Type_Vector (x-z and y-z plane faces)

Yellowstone Strong Scalability Results

University of Wyoming

Following results are computed in parallel on Intel Xeon E5-2670 processors with a clock speed of 2.6Ghz and 2GB per core memory.
•Yellowstone TAU benchmark = 8.4 sec

Yellowstone Specs:
• 1.504-petaflops peak IBM iDataPlex cluster
• 2.6-GHz Intel Xeon E5-2670 (Sandy Bridge) processors with Advanced Vector Extensions (AVX), 8 flops per clock (72,576)
• 144.58 TB total system memory

Results: Strong Scalability P = 4

Mesh Size: Nx=128, Ny=128, NZ=128
DOF(total)= 1,310,720,000
DOF=(number of fields)$(p+1)^3 NxNyNz$

Yellowstone Strong Scaling Results: P = 4

# Procs	DOF per Proc	Efficiency
1024*	1,280,000	1.0000
2048	640,000	0.9801
4096	320,000	0.9327
8192	160,000	0.9037
16384	80,000	0.8358

*Assumed Perfect

Results: Strong Scalability P = 7

Mesh Size: Nx=128, Ny=128, NZ=128
DOF(total)= 5,368,709,120
DOF=(number of fields)$(p+1)^3 NxNyNz$

Yellowstone Strong Scaling Results: P = 7

# Procs	DOF per Proc	Efficiency
1024*	5,242,880	1.0000
2048	2,621,440	0.9923
4096	1,310,720	0.9793
8192	665,360	0.9580
16384	327,680	0.9210

*Assumed Perfect

Results: Strong Scalability P = 9

Mesh Size: Nx=128, Ny=128, NZ=128
DOF(total)= 10,485,760,000
DOF=(number of fields)$(p+1)^3 NxNyNz$

Yellowstone Strong Scaling Results: P = 9

# Procs	DOF per Proc	Efficiency
1024*	10,240,000	1.0000
2048	5,120,000	0.9877
4096	2,560,000	0.9768
8192	1,280,000	0.9633
16384	640,000	0.9340

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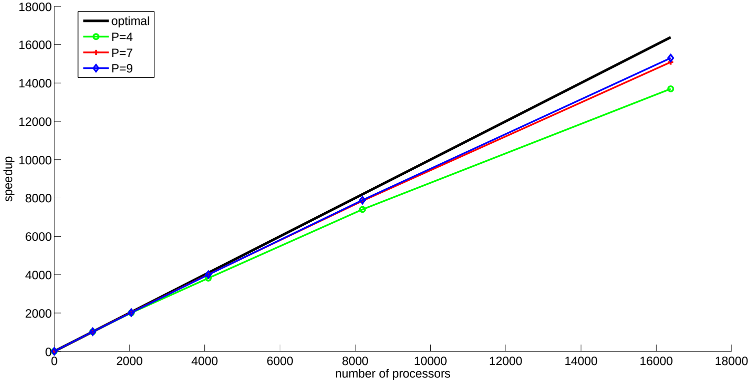
Results: Parallel Scalability



Future Work: Short Term Goals

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    - analogous version of SAMARC
    - different solution orders on different blocks

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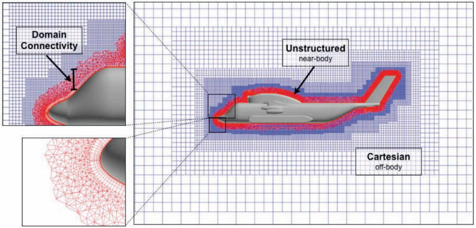
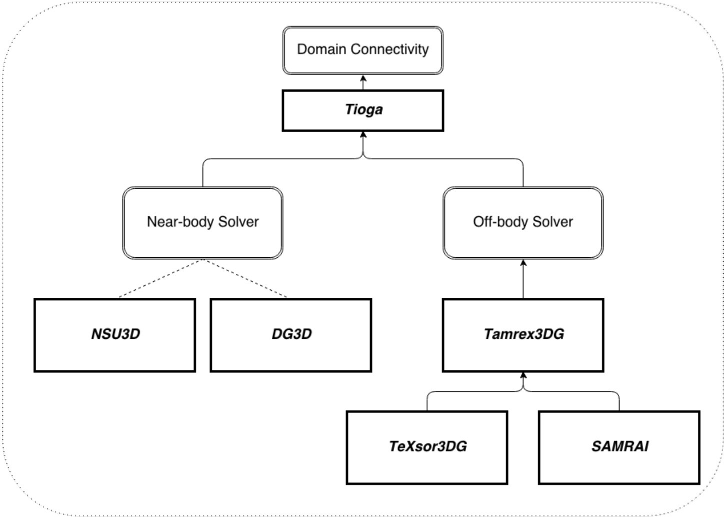
  - analogous version of HELIOS



Future Work: Short Term Goals cont.

3D Navier-Stokes Dual Mesh/Dual Flow Solver



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- Newton Implicit Solver
- Discrete Adjoint for design optimization and driver for AMR
- GPGPU version of TeXsor3DG

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Andy Wissink-Army Aviation Development Directorate

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- University of Wyoming Advanced Research Computing Center
- NASA Ames

Questions?